

Multiverse in the Third Quantized Formalism

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Abstract

In this paper we will analyze the third quantization of gravity in path integral formalism. We will use the time-dependent version of Wheeler-DeWitt equation to analyze the multiverse in this formalism. We will propose a mechanism for baryogenesis to occurs in the multiverse, without violating the baryon number conservation.

1 Introduction

The cosmological asymmetry between matter and antimatter in our universe is thought to be caused by the process of baryogenesis. In this process, the baryon number is not conserved [1]. In Grand Unified Theories (GUT), the baryon number conservation is violated by interactions of gauge bosons and leptiquarks. This forms the basis of all models of GUT baryogenesis [2, 3, 4, 5]. In this paper we will explain baryogenesis without violating baryon number conservation in the multiverse but only effectively violating it in particular universes. The existence of the multiverse first appeared in the many-worlds interpretation of quantum theory [6]. This idea has now resurfaced in the landscape of string theory [7, 8]. As this landscape is populated by 10^{500} vacuum states [9], the possibility of all of them being real vacuum states for different universes remains an open one [10]. As a matter of fact, this model of the multiverse has also been used as an explanation for inflation in chaotic inflationary multiverse [11]. It is a well-known fact that second quantization is more suited to studying many particle systems as compared to first quantization. Similarly, third quantization [13, 14] is naturally suited to studying many universes i.e., the multiverse [17, 18]. It may be noted that the third quantization of the Brans-Dicke theories [15] and Kaluza-Klein theories [16] have been already thoroughly studied.

In this paper we will first review the generalization of the wave function of the universe [19] to the time-dependent case [20, 21]. We will then analyse the multiverse in this model. In most model of the multiverse, universes do not interact with each other. However, in the models of multiverse that are used to study spacetime foam, baby universes branch off from a parent universe [23, 24, 25, 26, 27]. These models have been used to study the topological changes in spacetime, and thus have been employed to analyse the existence of the cosmological constant [28, 29, 30, 31]. Thus, these models are suited to study the formation of our universe from a collision of two previous universes. In these models, interactions are introduced, and different universes interact

with each other at Planck scale [32, 33]. In this paper, we will use this model of interacting universes to explain the occurrence of baryogenesis without violating the baryon number conservation. This will be done by assuming that an initial universe split into two different universes. The sum total of the baryon numbers in both the universes is conserved, but one universe can have more matter if the other one has exactly the same amount of extra anti-matter. Thus, the baryon number gets violated in individual universes without getting violated in the full multiverse. It may be noted that a similar analysis has been performed in minisuperspace models for Horava-Lifshits gravity [34]. As the third quantized universes can have their own conserved third quantized charges, a similar analysis has been performed in the fourth quantized formalism [35].

2 Time Dependent Wheeler-DeWitt Equation

In this section we will review the time-dependent wave function for the universe, which is a solution to the time-dependent Wheeler-DeWitt equation [20, 21]. The wave function of the universe can be obtained from the no-boundary proposal as follows [19]

$$\Psi[h_{ij}, \tilde{\phi}, \tilde{A}] = - \int Dg D\phi DA \mu[g, \phi, A] \exp(iS[g, \phi, A]), \quad (1)$$

Here, g denotes the metric, ϕ denotes all the matter fields and A denotes the all gauge fields. This integral is taken over all geometries with a compact boundary on which the induced metric is h_{ij} . Furthermore, the induced value of the matter and gauge fields on this boundary are denoted by $\tilde{\phi}$ and \tilde{A} , respectively. The invariance of the measure μ under an infinitesimal translation of N leads to the invariance of the wave function under the same. From this the Wheeler-DeWitt equation follows as

$$\frac{\delta \Psi}{\delta N} = -i \int Dg D\phi DA \mu[g, \phi, A] \frac{\delta S}{\delta N} \exp(iS). \quad (2)$$

Thus we get,

$$H\phi = 0, \quad (3)$$

where

$$H = h^{1/2} \left[-h^{-1/2} G_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} + R^{(3)}(h) - 2\Lambda - \frac{16\pi}{M_P^2} T_{nn} \left(-i \frac{\delta}{\delta \tilde{\phi}}, \tilde{\phi}, \tilde{A}, -i \frac{\delta}{\delta \tilde{A}}, \right) \right] \Psi[h_{ij}, \tilde{\phi}, \tilde{A}] = 0. \quad (4)$$

Here, G_{ijkl} is the metric on the superspace,

$$G_{ijkl} = \frac{1}{2} h^{-1/2} (h_{ik} h_{jl} + h_{il} h_{jk} - h_{ij} h_{kl}), \quad (5)$$

and

$$K_{ij} = \frac{1}{N} \left[-\frac{1}{2} \frac{\partial h_{ij}}{\partial t} + N_{(i|j)} \right]. \quad (6)$$

The scalar curvature $R^{(3)}$ is constructed from h_{ij} , and the covariant derivative with respect to the later quantity is represented by a stroke. The projection of

the stress-energy tensor for the matter and gauge fields normal to the surface is denoted by T_{nn} . We obtain the momentum constraint by following a similar procedure,

$$\frac{\delta\Psi}{\delta N_i} = -i \int Dg D\phi DA \mu[g, \phi, A] \frac{\delta S}{\delta N_i} \exp(iS). \quad (7)$$

Thus we get,

$$H_i \Psi = 0, \quad (8)$$

In the multiverse, the wave function of the universe would change. Thus, we would expect that the wave function of the universe to be analogous to a wave function for the time-dependent Schroedinger equation. Therefore, we want to obtain a time-dependent version of the Wheeler-DeWitt equation. It may be noted that in quantum mechanics a wave function can be constructed by using the path integral formalism

$$\psi(\vec{x}, t) = - \int D\vec{x}(t) \exp[iS(\vec{x}(t))]. \quad (9)$$

The Schroedinger equation can be obtained from this expression for the wave function

$$i \frac{\partial \psi}{\partial t} = H \psi. \quad (10)$$

This is because, we can write

$$\frac{\partial \psi}{\partial t} = -i \int D\vec{x}(t) \frac{\partial S}{\partial t} \exp(iS). \quad (11)$$

Now, a time-dependent version of the Wheeler-DeWitt equation can be constructed by following a similar procedure. Thus, the time-dependent wave function of the universe can define as [20, 21],

$$\Psi[h_{ij}, \tilde{\phi}, \tilde{A}] = - \int Dg D\phi DAM[g, \phi, A] \exp(iS[g, \phi, A]), \quad (12)$$

where, again g denotes the metric, ϕ denotes all the matter fields, A denotes the all gauge fields. The integral again is taken over all geometries with a compact boundary. However, now the measure factor $M[g, \phi, A]$ breaks the time translational invariance of the path integral. This in turn makes the wave function of the universe time-dependent. A possible choice for the measure, which does not change the quantum mechanical equation arising from the momentum constraint, $H_i \Psi = 0$, can be written as

$$M[g, \phi, A] = \mu[g, \phi, A] N^b. \quad (13)$$

This measure bring an explicit time dependence, but retains the the invariance of the spatial three-geometry even at quantum level. We can now write

$$\begin{aligned} \frac{\delta\Psi}{\delta N} &= - \int Dg D\phi DA \frac{\delta M}{\delta N} \exp(iS) \\ &\quad - i \int Dg D\phi DAM[g, \phi, A] \frac{\delta S}{\delta N} \exp(iS). \end{aligned} \quad (14)$$

The time-dependent version of the Wheeler-DeWitt equation obtained from this definition of the wave function can be written as

$$-i \frac{\delta\Psi}{\delta N} + H \Psi = 0. \quad (15)$$

3 Third Quantization

Now the wave function depends on the full metric $\Psi(g, \phi, A)$. So we can write the partition function for the multiverse as

$$Z_0 = \int D\Psi[g, \phi, A] \exp(iS[\Psi(g, \phi, A)]), \quad (16)$$

where

$$\begin{aligned} S[\Psi(g, \phi, A)] &= \int Dg D\phi DAM[g, \phi, A] \Psi(g, \phi, A) \\ &\times \left(-i \frac{\delta \Psi}{\delta N} + H \Psi \right) \Psi(g, \phi, A), \end{aligned} \quad (17)$$

is a free action for the multiverse. We can now define generating function for universes as follows

$$Z_0[J] = \int D\Psi \exp(iS[\Psi] + iJ\Psi), \quad (18)$$

where

$$J\Psi = \int Dg D\phi DAM[g, \phi, A] J(g, \phi, A) \Psi(g, \phi, A). \quad (19)$$

Now if we rotate N in complex plane as $N \rightarrow iN$, then we have

$$Z_0[J] = \int D\Psi \exp(-S[\Psi] - J\Psi), \quad (20)$$

This can now be expressed as

$$\begin{aligned} Z_0[J] &= \int Dg_1 Dg_2 D\phi_1 D\phi_2 DA_1 DA_2 M[g_1, \phi_1, A_1] M[g_2, \phi_2, A_2] \\ &\times \exp(J(g_1, \phi_1, A_1) \Delta[(g_1, \phi_1, A_1), (g_2, \phi_2, A_2)] \\ &\times J(g_2, \phi_2, A_2)), \end{aligned} \quad (21)$$

where

$$\Delta[(g_1, \phi_1, A_1), (g_2, \phi_2, A_2)] = \left[\text{Det} \left(-i \frac{\delta \Psi}{\delta N} + H \right) \right]^{-1}. \quad (22)$$

Now we define

$$\begin{aligned} &\left(-i \frac{\delta \Psi}{\delta N} + H \right) \Delta[(g_1, \phi_1, A_1), (g_2, \phi_2, A_2)] \\ &= \delta(g_1, g_2) \delta(\phi_1, \phi_2) \delta(A_1, A_2), \end{aligned} \quad (23)$$

where

$$\begin{aligned} &\int Dg_1 D\phi_1 DA_1 M[g_1, \phi_1, A_1] \delta(g_1, g_2) \delta(\phi_1, \phi_2) \\ &\times \delta(A_1, A_2) \Psi(g_1, \phi_1, A_1) \\ &= \Psi(g_2, \phi_2, A_2). \end{aligned} \quad (24)$$

Now we can written

$$\Delta[(g_1, \phi_1, A_1), (g_2, \phi_2, A_2)] = \frac{\delta^2 Z[J]}{\delta J(g_1, \phi_1, A_1) \delta J(g_2, \phi_2, A_2)} \Big|_{J=0}. \quad (25)$$

This is the amplitude for the wave function of a universe to change in the multiverse.

In most model of spacetime foam, baby universes interact with the parent universe at the Plank scale [28, 29]. The interaction term for a model of interacting universes has been analysed for time-independent Wheeler-DeWitt equation in third quantization formalism [32, 33]. We will generalize this model to time-dependent Wheeler-DeWitt equation and analyse some natural consequences of it. Thus, following on from the previous work on spacetime foam, we add the following interaction to the free action,

$$\begin{aligned} S_{int} = & \lambda \int Dg_3 Dg_1 Dg_2 D\phi_3 D\phi_1 D\phi_2 DA_3 DA_1 DA_2 \\ & M[g_1, \phi_1, A_1] M[g_2, \phi_2, A_2] M[g_3, \phi_3, A_3] \\ & \times (\Psi(g_1, \phi_1, A_1) \Psi(g_2, \phi_2, A_2) \Psi(g_3, \phi_3, A_3) \\ & \delta(g - g_1) \delta(g - g_2) \delta(\phi - \phi_1) \delta(\phi - \phi_2) \delta(A - A_1) \\ & \delta(A - A_2)). \end{aligned} \quad (26)$$

We can now define generating function for universes as follows

$$Z[J] = \int D\psi \exp(-S_t[\Psi] - J\Psi), \quad (27)$$

where

$$S_t[\Psi] = S[\Psi] + S_{int}[\Psi]. \quad (28)$$

So we have

$$Z[J] = \int D\Psi \exp(-S_t[\Psi] - J\Psi). \quad (29)$$

Now we get

$$Z[J] = \frac{\exp(-S_{int}Z_0[J])}{(\exp(-S_{int}Z_0[J])) \Big|_{J=0}}, \quad (30)$$

where

$$S_{int}[\lambda\Psi^3] = S_{int} \left[\lambda \left(-\frac{\delta}{\delta J} \right)^3 \right]. \quad (31)$$

In fact, as we can only observe the universes that interact with our universes, we have to use the generating function for connected universes only. This can be written as

$$W[J] = -\ln Z[J] \quad (32)$$

We can calculate the amplitude for a universe U_1 to split into two universes U_2 and U_3 . Now if $\Psi(g_1, \phi_1, A_1)$ is the wave function for U_1 , $\Psi(g_2, \phi_2, A_2)$ is the wave function for U_2 and $\Psi(g_3, \phi_3, A_3)$ for U_3 , then this amplitude is given by

$$\begin{aligned} & G[(g_1, \phi_1, A_1), (g_2, \phi_2, A_2), (g_3, \phi_3, A_3)] \\ = & \frac{\delta^3 W[J]}{\delta J(g_1, \phi_1, A_1) \delta J(g_2, \phi_2, A_2) \delta J(g_3, \phi_3, A_3)} \Big|_{J=0} \end{aligned}$$

$$\begin{aligned}
&= \lambda \int Dg D\phi DAM[g, \phi, A] \Delta[(g_1, \phi_1, A_1), (g, \phi, A)] \\
&\quad \times \Delta[(g, \phi, A), (g_2, \phi_2, A_2)] \Delta[(g, \phi, A), (g_3, \phi_3, A_3)] \\
&\quad + \mathcal{O}(\lambda^2).
\end{aligned} \tag{33}$$

The splitting of U_1 into U_2 and U_3 will appear as a big bang in U_2 and U_3 . This can also be used to possibly explain the domination of matter over antimatter in our universe without violating baryon number conservation [22]. Now, if the baryon number is conserved in U_1 , then we have

$$Bn_1 - Bm_1 = 0, \tag{34}$$

where Bn_1 represents the total baryon number of the baryons, Bm_1 represent the total baryon number of the anti-baryons in the universe U_1 . Here n_1 are the total number of baryons and m_1 are the total number of anti-baryons in universe U_1 . Now we represent the total number of baryons and in the universes U_2 as n_2, m_2 and U_3 as n_3, m_3 , respectively. The baryon number conservation implies

$$Bn_2 + Bn_3 - Bm_2 - Bm_3 = 0. \tag{35}$$

However, the baryon number in the multiverse does not constraint the baryon numbers in the universes U_2 or U_3 to be separately conserved. So, we can write

$$\begin{aligned}
Bn_2 - Bm_2 &\neq 0, \\
Bn_3 - Bm_3 &\neq 0.
\end{aligned} \tag{36}$$

Thus, after splitting of the universe U_1 , the universe U_2 can have more matter than anti-matter, if the universe U_3 has more anti-matter than matter. The extra matter in the U_2 will exactly balance the extra anti-matter in U_3 . Thus, the baryon number will still be conservation in the multiverse.

4 Conclusion

In this paper, we first derive an time-dependent version of the Wheeler-DeWitt equation. Then we analysed a classical Lagrangian density whose variation generates this equation. We also analysed the third quantization of this model. The occurrence of baryogenesis was explained due to the splitting of a earlier universe into two universes. This caused a effective violation of the baryon number conservation in individual universes, but it was still conserved in the multiverse.

It may be noted that we we arbitrary fixed the form of interactions in our model. It would be nice to derive the form of interaction from some underlying physical principle. One such model that can be studied is a third quantized gauge theory. We can construct the third quantized action from complex third quantized fields instead of real ones. If we do that, the potential part of the third quantized theory will be invariant under a gauge transformation. However, the kinetic part of the third quantized action can also be made gauge invariant by replacing all the second quantized functional derivatives by second quantized covariant functional derivatives. These covariant functional derivatives can be constructed by defining a third quantized gauge field whose gauge transformations exactly cancel the extra pieces generated by the action of the second

quantized functional derivatives on the third quantized field, in the third quantized action. We can then add a potential term for the third quantized gauge field thus introduced. The third quantized covariant derivatives will couple the third quantized fields to these gauge fields, and the interaction thus generated can also account for the baryogenesis. It would be interesting to analyse this model further.

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